Model Predictive Control of a Parafoil and Payload System

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I. Introduction

PARAFOIL and payload systems are lightweight, fly at low speed, provide soft landing capability, and are compact before deployment. The dynamics are sufficiently slow so that expert paraglider pilots can track a desired trajectory and attain accurate ground impact. Subconsciously, these pilots continuously project the trajectory forward in time and compare the results with the desired path. The error between the projected and desired path is used to determine control action. A control strategy that mimics how human pilots control paragliders is model predictive control. In model predictive control, a dynamic model of the system is used to project the state into the future and subsequently use the estimated future states to determine control action. It is a common control technique in the process control industry. The basic theory is detailed by Iko.

Nomenclature

\[ \begin{align*}
\tilde{b}, \tilde{c}, \tilde{d} & = \text{parasail span, parasail chord, and control flap width} \\
C_{D0}, C_{D2}, C_{Dha} & = \text{aerodynamic drag coefficients for parafoil and payload} \\
C_{L0}, C_{L2}, C_{Lha} & = \text{aerodynamic lift coefficients for parafoil and payload} \\
C_{q0}, C_{q2}, C_{qha} & = \text{aerodynamic roll coefficients for parafoil and payload} \\
C_{n0}, C_{n2}, C_{nha} & = \text{aerodynamic pitch coefficients for parafoil and payload} \\
F_A & = \text{aerodynamic force components in body reference frame} \\
F_W & = \text{combined weight force components of parafoil and payload in body frame} \\
H_p & = \text{prediction horizon} \\
H_T & = \text{inertia matrix of combined parafoil and payload system with respect to its mass center} \\
m_T & = \text{combined mass of payload and parasail} \\
p, q, r & = \text{components of angular velocity of system in body reference frame} \\
S_{ao} & = \text{skew symmetric cross-product operator of parafoil and payload system angular velocity} \\
T & = \text{transformation matrix from inertial to body reference frame} \\
V_A & = \text{total aerodynamic velocity of parafoil and payload system} \\
x, y, z & = \text{components of position vector of mass center in inertial frame} \\
\dot{x}, \dot{y}, \dot{z} & = \text{components of velocity vector of mass center in inertial frame} \\
\delta_{bias} & = \text{control bias} \\
\end{align*} \]

\[ \begin{align*}
\sigma & = \text{intersect parameter defining second point in desired path} \\
\phi, \theta, \psi & = \text{Euler roll, pitch, and yaw angles} \\
\end{align*} \]

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a recursive weighted least-squares method. Performance of the autonomous flight control system is shown through flight tests of the system under a variety of conditions.

II. Model Predictive Control

Consider a discrete system described in state-space form:

\[ x_{k+1} = Ax_k + Bu_k + D, \quad y_k = Cx_k \]  

(1)

Assume that the system matrices \( A, B, C, \) and \( D \) are known and that \( x_k \) is the state vector, \( u_k \) is the control input, and \( y_k \) is the output at time \( k \). The discrete model can be used to estimate the future state of the system. Under the assumption that a desired trajectory is known \((u_k)\), an estimated error signal \( \tilde{e}_k = y_k - \hat{y}_k \) is computed over a finite set of future time instants called the prediction horizon \( H_p \). The tilde is used to represent an estimated quantity. In model predictive control, the control computation problem is cast as a finite time discrete optimal control problem. To compute the control input at a given time instant, a quadratic cost function is minimized through the selection of the control history over the control horizon. The cost function can be written as

\[ J = (W - \tilde{Y})^T (W - \tilde{Y}) + U^T RU \]  

(2)

where

\[ W = \begin{bmatrix} w_{k+1} & w_{k+2} & \cdots & w_{k+H_p} \end{bmatrix}^T \]  

(3)

\[ \tilde{Y} = K_{CA} x_k + K_{CAB} U + K_{CAD} \]  

(4)

\[ U = \begin{bmatrix} u_k & u_k & \cdots & u_k & u_{k+H_p-1} \end{bmatrix}^T \]  

(5)

and \( R \) is a symmetric positive semidefinite matrix of size \( H_p \):

\[ K_{CA} = \begin{bmatrix} CA & C A^2 & \cdots & C A^{H_p} \end{bmatrix} \]  

(6)

\[ K_{CAB} = \begin{bmatrix} CB & 0 & 0 & 0 & 0 \\ CAB & CB & 0 & 0 & 0 \\ CA^2B & CAB & CB & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{H_p-1}B & \cdots & CA^2B & CAB & CB \end{bmatrix} \]  

(7)

\[ K_{CAD} = \begin{bmatrix} CD & CAD & CAD & CD \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \]  

(8)

Equation (4) is substituted into Eq. (2), resulting in the cost function in Eq. (9) that is in terms of the system state \( x_k \), desired trajectory \( W \), control vector \( U \), and system matrices \( A, B, C, \) and \( D \), and \( R \),

\[ J = (W - K_{CA} x_k - K_{CAB} U - K_{CAD})^T (W - K_{CA} x_k) \]  

\[ - K_{CAB} U - K_{CAD} + U^T RU \]  

(9)

The control \( U \), which minimizes Eq. (9), is

\[ U = K (W - K_{CA} x_k - K_{CAB}) \]  

(10)

where

\[ K = (K_{CAB}^T K_{CAB} + R)^{-1} K_{CAB}^T \]  

(11)

Equation (10) contains the optimal control inputs over the entire control horizon; however, at time \( k \), only the first element \( u_k \) is needed. The first element \( u_k \) is extracted from Eq. (10) by defining \( K_1 \) as the first row of \( K \). The optimal control over the next time sample becomes

\[ u_k = K_1 (W - K_{CA} x_k - K_{CAD}) \]  

(12)

where calculation of the first element of the optimal control sequence requires the desired trajectory \( W \) over the prediction horizon and the current state \( x_k \).

III. Parafoil and Payload Model

The combined system of the parafoil canopy and the payload is represented with six DOF, including three inertial position components of the system mass center as well as the three Euler orientation angles of the parafoil and payload system. Kinematic equations of motion for the parafoil and payload system are

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \]  

(13)

\[ \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = T \begin{bmatrix} p \\ q \\ r \end{bmatrix} \]  

(14)

The matrix \( T \) represents the transformation matrix from an inertial reference frame to the body reference frame:

\[ T = \begin{bmatrix} c_{\phi} c_{\theta} & c_{\phi} s_{\theta} & -s_{\phi} \\ c_{\phi} s_{\psi} c_{\theta} - s_{\phi} s_{\psi} & c_{\phi} s_{\psi} s_{\theta} + c_{\phi} c_{\psi} & c_{\phi} c_{\theta} \\ s_{\phi} s_{\psi} c_{\theta} + s_{\phi} s_{\psi} & s_{\phi} s_{\psi} s_{\theta} - s_{\phi} c_{\psi} & c_{\phi} c_{\theta} \end{bmatrix} \]  

(15)

The common shorthand notation for trigonometric functions is employed where sin(\(\alpha\)) \(\equiv s_\alpha\), cos(\(\alpha\)) \(\equiv c_\alpha\), and tan(\(\alpha\)) \(\equiv t_\alpha\). The dynamic equations of motion are

\[ \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{1}{m_T} (F_A + F_W) - T S_u T^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \]  

(16)

\[ \begin{bmatrix} \dot{q} \\ \dot{r} \end{bmatrix} = I_T^{-1} \left( M_A - S_u I_T \begin{bmatrix} p \\ q \end{bmatrix} \right) \]  

(17)

where

\[ S_u = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \]  

(18)

\[ I_T = \begin{bmatrix} I_{XX} & 0 & I_{XZ} \\ 0 & I_{YY} & 0 \\ I_{SZ} & 0 & I_{ZZ} \end{bmatrix} \]  

(19)

\[ I_T^{-1} = \begin{bmatrix} I_{XX} & 0 & I_{XZ} \\ 0 & I_{YY} & 0 \\ I_{SZ} & 0 & I_{ZZ} \end{bmatrix} \]  

(20)

The weight force vector in the body reference frame is

\[ F_W = m_T R \begin{bmatrix} -s_\phi \\ s_\phi c_\theta \\ c_\phi c_\theta \end{bmatrix} \]  

(21)

The aerodynamic forces acting at the system mass center and the aerodynamic moments about the system mass center are given in Eqs. (22) and (23), respectively. \( C_{pl} \) is introduced to eliminate the center of pressure location from the dynamic equations while maintaining the parafoils tendency to glide with no roll during neutral control.
If a reduced state $\delta_p$ be controlled. The desired states to control in a parafoil and pay-
limiting to nearly straight flight and is not changes about a nominal yaw angle. Constraining the yaw angle
of the nonlinear model must constrain the yaw angle to only small linear six-DOF model that accurately represents the inertial position
not appear in any of the equations of motion. However, yaw angle
and Euler yaw angle reach a steady state. The inertial positions do
it falls. All of the states excluding the inertial positions
apply standard model predictive control, must be linearized. Con-
describing the parafoil and payload system are nonlinear and, to
that the aerodynamic velocity
rolling and pitching in Eqs. (14) and (17) can be linearized assuming

$$\dot{\delta} = \begin{bmatrix}
\delta
\psi
\rho
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} + \frac{1}{c_0} + \frac{\rho \bar{S} \bar{b} V_A^2 I_{XXI} C_{\phi}}{2} + \frac{\rho \bar{S} \bar{b}^2 V_A^4 I_{XXI} I_{XXI} C_{\phi}}{4} + \frac{\rho \bar{S} \bar{b}^2 V_A^2 I_{XZI} C_{\phi}}{2} + \frac{\rho \bar{S} \bar{b}^2 V_A^4 I_{XZI} C_{\phi}}{4} \begin{bmatrix}
\delta
\psi
\rho
\end{bmatrix}
$$

$$A typical desired trajectory of a parafoil and payload system consist of points in the $x$–$y$ plane, and, according to Eq. (1), the desired output must be a linear combination of the linear model states. To use the linear model described in Eq. (24) for model predictive control, the desired trajectory in the $x$–$y$ plane must be mapped into a desired trajectory in terms of the reduced states $[\phi \, \psi \, p \, r]^T$. A straightforward mapping is to assume that the side velocity $v$ is small, that the parafoil is traveling in the direction of its heading $\psi$, and that the forward velocity is constant. A desired path defined by points can then be converted to desired heading angle using parametric Lagrange interpolating polynomials.

### IV. Test System

The parafoil and payload system used in all testing is shown in Figs. 1 and 2 with the physical parameters in Table 1. A test flight commences by launching the system from the ground. A 10-in. pro-
eller powers the test system to altitudes of from 250 to 400 ft,
whereas the parafoil is stopped and gliding commences, lasting ap-
proximately 20 s for every 100 ft of altitude.

Full-state measurement of the parafoil required in the optimal control sequence is achieved through a sensor package that includes three single-axis gyroscopes, a three-axis accelerometer, and a three-axis magnetometer. Inertial positions $x$ and $y$ required in the mapping of the desired $x$–$y$ path into a desired yaw angle are obtained from a wide area augmentation system enabled global po-
sitioning satellite receiver. The sensors are supplemented with a wireless receiver that transmits data from the parafoil and re-
ceives commands during flight. An operator-controlled transmitter switches control of the parafoil to one of three modes: manual, esti-
mation, or autonomous. Manual mode allows the operator to fly the parafoil manually. Estimation mode allows estimation of linear

![](image-url)

**Fig. 1 Payload.**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>$\rho$</td>
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<td>slug/ft$^3$</td>
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<tr>
<td>Weight</td>
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<td>lbf</td>
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<tr>
<td>$S$</td>
<td>7.5</td>
<td>ft$^2$</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>4.25</td>
<td>ft</td>
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<tr>
<td>$\bar{d}$</td>
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<td>ft</td>
</tr>
<tr>
<td>$I_{XX}$</td>
<td>0.1357</td>
<td>slug/ft$^2$</td>
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<tr>
<td>$I_{YY}$</td>
<td>0.1506</td>
<td>slug/ft$^2$</td>
</tr>
<tr>
<td>$I_{ZZ}$</td>
<td>0.0203</td>
<td>slug/ft$^2$</td>
</tr>
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<td>ft$^2$/slug</td>
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<td>ft$^2$/slug</td>
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<td>ft$^2$/slug</td>
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<td>$I_{XX1}$</td>
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<td>ft$^2$/slug</td>
</tr>
<tr>
<td>$V_A$</td>
<td>21.6</td>
<td>ft/s</td>
</tr>
</tbody>
</table>

Table 1 Parafoil and payload physical parameters

---

A typical desired trajectory of a parafoil and payload system consist of points in the $x$–$y$ plane, and, according to Eq. (1), the desired output must be a linear combination of the linear model states. To use the linear model described in Eq. (24) for model predictive control, the desired trajectory in the $x$–$y$ plane must be mapped into a desired trajectory in terms of the reduced states $[\phi \, \psi \, p \, r]^T$. A straightforward mapping is to assume that the side velocity $v$ is small, that the parafoil is traveling in the direction of its heading $\psi$, and that the forward velocity is constant. A desired path defined by points can then be converted to desired heading angle using parametric Lagrange interpolating polynomials.
model aerodynamic coefficients required for model predictive control. Autonomous mode controls the parafoil using the model predictive control law.

V. Identification of Aerodynamic Coefficients

Application of the reduced-order model requires knowledge of five constant aerodynamic coefficients, $C_{1\rho}$, $C_{lp}$, $C_{lha}$, $C_{ar}$, and $C_{nha}$, and the constant bias term $\delta_{bias}$. The six parameters are estimated using recursive weighted least-squares estimation, where $\delta$ are measurements, $x_i$ are parameters to be estimated, and $n_i$ is zero mean measurement noise:

$$ z_i = H_i x + n_i $$

The recursive weighted least-squares estimation to Eq. (25) is given in Eqs. (26) and (27), where $P_i$ is the error covariance estimate of the parameters at measurement $i$ and $Q$ is the measurement noise covariance:

$$ \hat{x}_i = \hat{x}_{i-1} + P_i H_i^T Q^{-1}(z_i - H_i \hat{x}_i) $$

$$ P_i = P_{i-1} - P_{i-1} H_i^T (Q + H_i P_{i-1} H_i^T)^{-1} H_i P_{i-1} $$

The matrix $H_i$ yields a linear relationship between the parameters to be estimated, and measurements are acquired by linearizing $\delta p$ and $\delta r$ in Eq. (24):

$$ H_i = \frac{\rho \delta b V_A^2}{2} \begin{bmatrix}
I_{XX} \phi_i & I_{XZ} \phi_i \\
I_{XX} & \frac{\hat{b}_p}{V_A} & I_{XZ} \\
I_{XX} & \frac{\hat{b}_r}{V_A} & I_{ZZ}
\end{bmatrix}^T $$

The recursive weighted least-squares estimation requires differentiation of measured roll and yaw rates. The control sequence used in parameter identification was chosen to be sinusoidal to ensure that numerical differentiation of roll and yaw rates produced significant signals. Measured roll and yaw rates are processed with a zero-phase digital filter before differentiation. The recursive weighted least-squares estimation is initialized with $P_i$ as a $6 \times 6$ diagonal matrix with 0.05 along the diagonal and $C_{1\rho}$, $C_{lp}$, $C_{lha}$, $C_{ar}$, $C_{nha}$, and $\delta_{bias}$ as $-0.1$, $-0.5$, $0.1$, $-0.1$, $0.1$, and 0.0, respectively. The measurement noise covariance $Q$ was set as a $2 \times 2$ diagonal matrix with $Q_{1,1} = 0.00475$ and $Q_{2,2} = 0.0005$. The estimated aerodynamic coefficients, $C_{1\rho}$, $C_{lp}$, $C_{lha}$, $C_{ar}$, $C_{nha}$, and $\delta_{bias}$ from the flight data are given in Table 2. The discrete time linear reduced-order model is verified by comparing simulated results using the estimated aerodynamic coefficients with measured flight data. Figure 3 shows that the reduced-order model is able to capture the fundamental dynamics of the parafoil and payload.

VI. Model Predictive Control Results

The prediction of desired heading angle with third-order Lagrange interpolating polynomials is accomplished using four desired path points. The first point is defined as the current position of the parafoil and payload system. The second point is defined as the location along the desired path that is a distance $\sigma$ ahead of the current position and is called the intercept parameter. The third and fourth points are the next two desired path points. Figure 4 shows a desired path and the Lagrange interpolating polynomial found. The update rate of the model predictive controller was chosen to be 1 s, and the linear model is converted to a discrete time system of the form in Eq. (1) with a sampling period of 1 s. The discrete time system matrices $A$, $B$, $C$, and $D$ required for the model predictive controller are

$$ A = \begin{bmatrix}
0.5243 & 0 & 0.5823 & 0.0100 \\
0.0120 & 1.0000 & 0.0125 & 0.1372 \\
-0.7589 & 0 & 0.1360 & 0.0047 \\
0.0056 & 0 & 0.0148 & 0.0009
\end{bmatrix} $$

Table 2 Estimated model coefficients

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{1\rho}$</td>
<td>$-0.0100$</td>
</tr>
<tr>
<td>$C_{lp}$</td>
<td>$-0.0520$</td>
</tr>
<tr>
<td>$C_{lha}$</td>
<td>$0.0021$</td>
</tr>
<tr>
<td>$C_{ar}$</td>
<td>$-0.0850$</td>
</tr>
<tr>
<td>$C_{nha}$</td>
<td>$0.0010$</td>
</tr>
<tr>
<td>$\delta_{bias}$</td>
<td>$-0.0001$</td>
</tr>
</tbody>
</table>
The matrix $R$ penalizing control magnitude in the optimal control sequence is selected as an $H_p \times H_p$ matrix with 0.35 on the diagonal and zeros everywhere else. Results for the model predictive controller are shown in Figs. 5–7 with $H_p = 10$ and $\sigma = 100$ ft. Figure 5 shows the measured path of the parafoil and payload compared to a desired straight path with no wind; the markers designate 5-s intervals with the first marker being the initial position at 0. The control sequence is shown in Fig. 8. Control is initiated with the parafoil and payload initially traveling away from the desired path and 40 ft offtrack. The parafoil has a maximum error of 75 ft at 100 ft downrange, then overshoots the desired path by 39 ft at $-510$ ft downrange before a final error of 9 ft at impact. Figure 6 shows the measured path of the parafoil and payload compared to the desired straight path and control with a 12-ft/s crosswind from positive to negative crossrange. Control is again shown in Fig. 8 and is initiated with the parafoil and payload initially traveling away from the desired path and 100 ft offtrack. The parafoil has a similar oscillatory response with a maximum error of 119 ft at 230 ft downrange as it overshoots the desired path. The parafoil turns back toward the desired path and comes within 18 ft before the wind pushes it farther away. The final error at impact is 6 ft. The larger error from the crosswind is due to the difference in measured yaw angle and heading angle because of parafoil sideslip. Figure 7 shows the performance of the model predictive controller when tracking the more complicated S-shaped path. Control is initiated when the parafoil and payload are 210 ft offtrack. The maximum error during the flight is 45 ft at 550 ft downrange and $-550$ ft crossrange. The model predictive controller is able to predict the required control input so that the parafoil and payload system are able to achieve close proximity to the desired points as they are passed.

VII. Conclusions

Model predictive control is a natural way to control a parafoil and payload because it mimics the process that a pilot controlling a paraglider estimates both the path and control sequence to achieve a desired outcome. The work reported here employs model predictive control for autonomous control of a parafoil and payload system. To support the flight control law, a reduced-state linear model was created that uses roll angle, yaw angle, body roll rate, and body yaw rate of the parafoil and payload system. Application of the reduced-order model requires knowledge of five constant aerodynamic coefficients, $C_{l_{\phi}}$, $C_{l_{p}}$, $C_{l_{\delta}}$, $C_{n_{\delta}}$, and $C_{n_{\phi}}$, and a constant bias term $\delta_{\text{bias}}$. A recursive weighted least-squares estimation is used to estimate the six parameters. The estimated parameters and reduced-state model is compared with flight data, and it is shown that they adequately model the parafoil and payload system. To use the reduced-state linear model, the desired $x$–$y$ trajectory is mapped into desired yaw angles using Lagrange interpolating polynomials assuming a constant aerodynamic velocity. Three exemplar autonomous flight tests are used to show that model predictive control is an effective way to control autonomously the trajectory of a parafoil and payload system.

References


